

2018 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIPS 2018

学科試験 問題
EXAMINATION QUESTIONS

(学部留学生)
UNDERGRADUATE STUDENTS

数 学 (B)
MATHEMATICS (B)

注意 ☆試験時間は 60 分。

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) If $x > 0$, $y > 0$, and $x + 3y = 2$, then $\frac{1}{xy} \geq$ with equality if and only if $x =$ and $y =$.

(2) The real-number solution to the equation $2^{x+2} - 2^{-x} + 3 = 0$ is $x =$.

(3) Let a, b be constants. If the polynomial $x^4 + ax^3 + ax^2 + bx - 6$ is divisible by $(x - 1)^2$, then $a =$, $b =$.

(4) The maximum value of the function $\sin 3x$ for $\frac{5}{18}\pi \leq x \leq \frac{2}{3}\pi$ is , and the minimum value of that is .

(5) For the complex number $z = \cos \frac{1}{3}\pi + \sqrt{-1} \sin \frac{1}{3}\pi$, the following equality holds: $z + z^2 + z^3 + z^4 + z^5 =$.

(6) Let S be a square with side length 1, T a triangle with side lengths 1, 1, $\sqrt{2}$, and R a triangle with side lengths 1, $\sqrt{2}$, $\sqrt{3}$. Let C be a pyramid with a base that is S and lateral faces consisting of two T 's and two R 's. The volume of C then is .

2. On the xy -plane, a circle with center (a, b) and radius r is tangent to the parabola $y = x^2$ at two distinct points. Fill in the blanks with the answers to the following questions.

- (1) When one of the two points of tangency is (t, t^2) , express a , b , and r in terms of t .
- (2) When one of the two points of tangency is $\left(\frac{1}{2}, \frac{1}{4}\right)$, express the area S of the finite region bounded by the circle and the parabola in terms of π .

(1) $a =$ $b =$

$r =$

(2) $S =$

3. Let $f(x) = \int_0^x \frac{1}{1+t^2} dt$ ($x > 0$), and $g(x) = f\left(\frac{1}{x}\right)$. Fill in the blanks with the answers to the following questions.

- (1) Compute the derivatives $f'(x) = \frac{d}{dx}f(x)$ and $g'(x) = \frac{d}{dx}g(x)$.
- (2) Compute the value of $f(1)$.
- (3) The function $f(x) + g(x)$ is a constant C . Find the value of C .
- (4) Calculate the limit $\lim_{x \rightarrow \infty} f(x)$.

(1) $f'(x) =$ $g'(x) =$

(2) $f(1) =$ (3) $C =$

(4) $\lim_{x \rightarrow \infty} f(x) =$